تم الرفع بواسطة
م. معن ابو عيسى

رياضيات هندسية (٢)
First

Palestine Technical University

44/100

Department of Applied Mathematics

Engineering Math II

First Exam, Summer 2011

Supervisor :Ata Asa'd **27/6/2011**

Student Name(بالعربية)...[redacted]. Time :1 hour

Question One: (20p)

Determine which of following statements true or false.

$ce^{-\int 1\,dt} = ce^{-t}$

1. (T)If Y^{iv}+Y=0 then W(y_1,y_2,y_3)= ce^{-t}
2. (F)one of the forth root for -1 is $w_0=e^{-i\frac{10\pi}{4}}$
3. (X)The solutions of XY''' -Y''=0 ,1 ,X, X^3 are linearly Independent.
4. (F)The Argument of $Z=-3-\sqrt{3}i$ is $\theta=-\frac{7\pi}{6}$.
5. (F)$x\sum_{n=0}^{\infty} a_n x^n = \sum_{n=1}^{\infty} a_{n-1}x^{n+1}$ is right always
6. (T)The interval in which the solution exist for I.V , tY''' +(sint)Y''+3y=cost ; y(-1)=3 is $(-\infty, 0)$
7. (F)A point X_0 such that $p(X_0)\neq 0$ is called singular point for D.E p Y''+qY'+rY=0,where p, q ,r are continues function
8. (T)The recurrence relation Y' - Y=0 is $a_{n+1}=\frac{a_n}{n+1}$
9. (F)The undetermined coefficient for the particular solution of Y''' -4Y' =cost is A cost+ Bsint
10. (T)If $Y_1=e^t$,$Y_2=-e^{-t}$ is a two particular solution then y=2cosht is the solution for the homogenous equation

14

$\exp i\left(\frac{\pi}{4}+\frac{2k\pi}{4}\right)$

$r-1=0 \quad r=1$

$y=c_1e^t$

$r^3-4r=0$

$r(r^2-4)=0$

$r=0 \Rightarrow r=\pm 2$

$c_1+c_2e^{2t}+c_3e^{-2t}$

e^t+e^{-t}

$a_{n+1}=\frac{a_n}{n+1}$

Question Two: (30p)

a) Solve the equation for the real number x , y (10p)

(3-2i)(x+iy)=3(x-2iy)+3i-1

$\cancel{3x}+3iy-2xi+2y=\cancel{3x}-6iy+3i-1$

$1-\sqrt{3}i$

b)find the forth roots for $1-\sqrt{3}\,i$ (10p)

$|B| = \sqrt{1+3} = \sqrt{4} = 2$

$\theta = \frac{yi}{x} = -\sqrt{3} \Rightarrow \theta = \frac{-\pi}{3}$

$2\exp i\left(\frac{-\pi}{3\times4} + \frac{2k\pi}{4}\right)$

$k_0 = 2\exp i\left(\frac{-\pi}{12}\right) \Rightarrow 2e^{\frac{-\pi}{12}i} \overset{w_0\,\frac{1}{4}}{=} 2\left(\cos\frac{-\pi}{12} + \sin\frac{-\pi}{12}i\right)$ — (1)

$k_1 = 2\exp i\left(-\frac{\pi}{12} + \frac{1}{2}\pi\right) = 2\exp i\left(\frac{5\pi}{12}\right) \overset{w_1\,\frac{1}{4}}{=} 2\left(\cos\frac{5\pi}{12} + \sin\frac{5\pi}{12}i\right)$ — (2)

$k_2 = 2\exp i\left(\frac{-\pi}{12} + \pi\right) = 2\exp i\left(\frac{11\pi}{12}\right) \Rightarrow w_2\ \frac{1}{4}2\left(\cos\frac{11\pi}{12} + \sin\frac{11\pi}{12}i\right)$

$k_3 = 2\exp i\left(-\frac{\pi}{12} + \frac{6\pi}{4}\right) = 2\exp i\left(-\frac{\pi}{12} + \frac{18\pi}{12}\right) = 2\exp i\left(\frac{17\pi}{12}\right) \Rightarrow w_3 = 2^{\frac{1}{4}}\left(\cos\frac{17\pi}{12} + \sin\frac{17\pi}{12}i\right)$

10

c) Use De Moiver's theorem to express the trigonometric function. (10p)

1)sin3t

$y = \sin 3t$

$y' = 3\cos 3t$

$y'' = -9\sin 3t$

$y'' - y' + 9y + 3\sin 3t = 0$

$-9\cos 3t + 3\sin 3t + 9\cos 3t + 3\sin 3t = 0$

2)cos3t

$y = \cos 3t$

$y' = -3\sin 3t$

$y'' = -9\cos 3t$

$r^3-2r^2-r+2=0$

Question Three: (25p)

Use variation of parameter to find the solution of

$Y''' - 2Y'' - Y' + 2y = e^{4t}$

$-2, -1$

$r^3 + 2r - r + 2 = 0$

$r_1 = r_2 = +2 \quad r_3 = -1$

$y = c_1 e^{-2t} + c_2 t e^{-2t} + c_3 e^{-t}$

$$W \begin{vmatrix} 1 & 1 & t \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix} = 1(0) - 1(0) + t(0) = 0$$

$$W_1 \begin{vmatrix} 0 & 1 & t \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 0(0) - 1(0-1) + t(0) = 1$$

هذا السؤال خلف الورقة

$$W_2 = \begin{vmatrix} 1 & 0 & t \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = 1(0-1) + 0 + t(0) = -1$$

$$W_3 = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1(0) - 1(0) - 0 = 0$$

$= 1 \int_0^t \underline{e^{4t} \cdot zero} = 0$

$y(s_1) = 1 \int_0^t e^{4t} 1 = \int_0^t e^{4t} = \frac{e^{4t}}{4} \Big|_0^t = \frac{e^{4t}}{4} - \frac{1}{4}$

$y(s_2) = -1 \int e^{4t} = -\int_0^t e^{4t} = -\frac{e^{4t}}{4} = -\left(\frac{e^{4t}}{4} - \frac{1}{4}\right) = -\frac{e^{4t}}{4} + \frac{1}{4}$

$y(s_3) = 0 \quad y(t) = 0 + \frac{e^{4t}}{4} - \frac{1}{4} + \frac{-e^{4t}}{4} + \frac{1}{4} = 0$

$y''' - 2y'' - y' + 2y = e^{4t}$

$r^3 - 2r^2 - r + 2 = 0$

$\Longrightarrow r_1 = r_2 = -2 \quad , r_3 = -1$

$y(t) = c_1 e^{-2t} + c_2 t e^{-2t} + c_3 e^{-t}$

$-2te^{-2t}$

$4te^{-2t} + e^{-2t} * -2$

$W = \begin{vmatrix} e^{-2t} & te^{-2t} & e^{-t} \\ -2e^{-2t} & -2te^{-2t} + e^{-2t} & \end{vmatrix}$

te^{-2t}

$-2te^{-2t} + e^{-2t} * 1 =$

$\begin{vmatrix} e^{-2t} & te^{-2t} & e^{-t} \\ -2e^{-2t} & -2te^{-2t} + e^{-2t} & -e^{-t} \\ 4e^{-2t} & 4te^{-2t} - 4e^{-2t} & e^{t} \end{vmatrix}$

$e^{-2t}\left(\left(-2te^{-2t} + e^{-2t} * e^{t}\right) - \left(4te^{-2t} - 2e^{-2t} * -e^{-t}\right)\right)$

$= e^{-2t}\left(\left(2te^{-t} + e^{-t}\right) - \left(-4te^{-3t} + 2e^{-3t}\right)\right)$

$= e^{-2t}\left(\left(2te^{-t} + e^{-t} + 4te^{-3t} - 2e^{-3t}\right)\right)$

$\boxed{= 2ete^{-3t} + e^{-3t} + 4te^{-5t} - 2e^{-5t}}$ 1

$\begin{vmatrix} 0 & te^{-2t} & e^{t} \\ 0 & -2te^{-2t} + e^{-2t} & -e^{-t} \\ 1 & 4te^{-2t} - 2e^{-2t} & e^{t} \end{vmatrix}$

$= 0 + te^{-2t}\left(0 + e^{-t}\right) +$

$+ e^{t}\left(-2te^{-2t} + e^{-2t} * e^{t}\right) - \left(4te^{-2t} - 2e^{-2t} * -e^{-t}\right)$

$= te^{-3t} + e^{t}\left(-2te^{-t} + e^{-t}\right) - \left(4te^{-3t} - 2e^{-3t}\right)$

$= te^{-3t} + \left(-2t + 1\right) - \left(4te^{-2t} - 2e^{-2t}\right)$ 2

10

$\begin{vmatrix} e^{-2t} & 0 & e^{-t} \\ -2e^{-2t} & 0 & -e^{-t} \\ 4e^{-2t} & 1 & e^{t} \end{vmatrix}$

Page 4

Question four: (25p)

Solve the given D.E by means of power series about the given point find the first fourth term in each of two linear independent solution.

$(1-x)Y'' + y = 0 \quad , x_0 = 0$

$(1-x)y'' + y = 0$

$y'' + y = 0$

$r^2 + 1 = 0$

$r = \pm i$

$y = \sum_{n=0}^{\infty} a_n x^n$

$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$

$y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$

$(1-x)\left(\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=0}^{\infty} a_n x^n\right.$

$= \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - \sum_{n=2}^{\infty} n(n-1) a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^n$

Shiffting

$\sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n - \sum_{n=0} (n+2)(n+1) a_{n+2} x^{n+1} + \sum_{n=0}^{\infty} a_n x^n$

$\Rightarrow$

$\sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} - (n+2)(n+1) a_{n+2} x^{n+1} + a_n x^n$

1 0